Kuwait University | Math 101 | Date: December 16, 2004
Dept. of Math. & Comp. Sci. | Second Exam | Duration: 75 minutes

Calculators, mobile phones, pagers and all other mobile communication equipment are not allowed

Answer the following questions:

1. Let $f(x) = \dfrac{\sqrt{3+x}}{x}$. Use differentials to approximate $\dfrac{\sqrt{3.9}}{0.9}$. (3 pts.)

2. Find equations of the normal lines to the graph of the equation

$$4x + y^2 + (xy)^2 = 6$$

at the point whose $x-$coordinate is 1. (4 pts.)

3. (a) State The Mean Value Theorem.

   (b) Suppose that f is continuous on $[a, b]$ and $f'(x) < 0$ for every $x \in (a, b)$. Use The Mean Value Theorem to show that $f(b) < f(a)$. (4 pts.)

4. Two students start walking from the same point. One walks south at a rate of 4 m/sec and the other walks west at a rate of 3 m/sec. At what rate is the distance between the two students increasing 2 seconds later. (4 pts.)

5. Let $f(x) = \dfrac{x}{x+1}$.

   (a) Find the vertical and horizontal asymptotes for the graph of f, if any.

   (b) Show that $f'(x) = \dfrac{1}{(x+1)^2}$. Find the intervals on which the graph of f is increasing and the intervals on which the graph of f is decreasing. Find the local extrema of f, if any.

   (c) Find the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward. Find the points of inflection, if any.

   (d) Sketch the graph of f.

   (e) Find the maximum and the minimum values of f on $[0, 3]$. (10 pts.)

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1. $f'(x) = \dfrac{-(6+x)}{2x^2\sqrt{3+x}} \implies \dfrac{\sqrt{3.9}}{0.9} = f(0.9) \approx f(1) + f'(1)(-0.1) = 2 + \left(-\dfrac{7}{4}\right)(-0.1) =$ $\boxed{2.175}$.

2. At $x = 1$, $y = \pm 1$. Differentiation with respect to $x \implies 4 + 2yy' + 2xy^2 + 2x^2yy' = 0$.
Equation of normal line at $P_1(1,1): y - 1 = \frac{2}{3}(x-1)$.
Equation of normal line at $P_2(1,-1): y + 1 = -\frac{2}{3}(x-1)$.

3. (b) $f'(x) < 0$ for every x in $(a,b) \implies f'$ exists on $(a,b) \implies f$ is differentiable on (a,b) and since f is continuous on $[a,b]$. From the M.V.T., $\exists\, c \in (a,b)$ such that: $f(b) - f(a) = f'(c)(b-a) < 0\ \{f'(c) < 0, b - a > 0\} \implies f(b) < f(a)$.

4. $z^2 = x^2 + y^2 \implies z = \sqrt{(3t)^2 + (4t)^2} \implies \dfrac{dz}{dt} = \dfrac{5t}{\sqrt{t^2}} \implies \left.\dfrac{dz}{dt}\right|_{t=1} = 5$ m/sec

5. $D_f = \mathbb{R} - \{-1\}$. The graph of f is not symmetric. $f(0) = 0$.
$f'(x) = \dfrac{(x+1)(1) - x}{(x+1)^2} = \dfrac{1}{(x+1)^2} = (x+1)^{-2} \implies f''(x) = \dfrac{-2}{(x+1)^3}$.
$\lim_{x \to -1^\pm} f(x) = \mp\infty \implies$ The graph of f has infinite discontinuity at $x = -1$ & $x = -1$ is V.A.
$\lim_{x \to \pm\infty} f(x) = 1 \implies y = 1$ is H.A.
$f'(-1)$ does not exist.

I	$(-\infty, -1)$	$(-1, \infty)$
sign of $f'(x)$	+	+
Conclusion	f is increasing on $(-\infty, -1)$.	f is increasing on $(-1, \infty)$.

$f''(-1)$ does not exist.

I	$(-\infty, -1)$	$(-1, \infty)$
sign of $f''(x)$	+	−
Concavity	The graph of f is CU on $(-\infty, -1)$	The graph of f is CD on $(-1, \infty)$

$f(0) = 0$ is the min. value of f on $[0,3]$. $f(3) = \frac{3}{4}$ is the max. value of f on $[0,3]$.